The Canadian Gravimetric Geoid Model of 2000 (CGG2000)

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Abstract: The Geodetic Survey Division, in collaboration with the University of Calgary and the University of New Brunswick, developed a new and enhanced gravimetric geoid model for Canada (CGG2000). This model replaces the GSD95 geoid model. CGG2000 was developed according to the three-year plan in which we recommend actions to be taken to improve theory, data set and computational process. The new geoid model follows the Helmert-Stokes scheme, i.e., the gravity measurements are reduced in terms of Helmert's second condensation method and the geoid heights are determined using the Stokes integral. All related reductions to the gravity measurements are determined using the spherical approximation. The under-laying global geopotential model is EGM96 (degree and order 360) and it contributes its long wavelengths up to degree and order 30 through the modified spheroidal Stokes kernel. The 1D-FFT procedure solves the Stokes integral. CGG2000 model is validated against GPS/Levelling across Canada. For 1090 benchmarks distributed across Canada, the mean and standard deviation is –0.260 m and 0.179 m, respectively. Part of the misfit is probably due to a systematic error in the Canadian primary levelling network.

1 Introduction

With the completion of the GSD95 geoid model (Véronneau, 1997), the Geodetic Survey Division (GSD) of Natural Resources Canada, in association with the University of New Brunswick (UNB) and the University of Calgary (UofC), prepared a three-year plan in view of the development of the next model. The three-year plan (Pagiatakis, 1996) states the actions to be taken by the three agencies regarding geoid theory, required data and computational process. The main objective is the development of a geoid model for Canada with accuracy of 1 cm from coast to coast. Even though the current data set might not allow us to reach such a precision, at least the theory is developed at that level.

The determination of a geoid model with accuracy of 1 cm will allow the modernization of heights determination through space techniques. For example, when the geoid model is combined with the technology of the Global Positioning System (GPS), it offers a cost-efficient approach versus the traditional levelling methods. In addition, the geoid model can be highly beneficial to oceanographers for the determination of the sea surface topography and ocean currents when it is combined with satellite altimetry data.

This paper is a review of the procedure used for the determination of the CGG2000 geoid model. Sections 2 to 7 give a summary of the theory, assumptions and approximations behind the new geoid model for Canada. The theory of CGG2000 is principally derived from *Martinec* (1993 and 1998) and *Vaníček et al.* (1999). Section 2 discusses Bruns' formula, i.e., the relation between potential and geoid height. The third section is the derivation of the Helmert anomalies on the geoid. In sections 4 and 5, we describe the approach used for the global evaluation of the Stokes integral. The procedure for the determination of the mean Helmert anomalies is mentioned in section 6. Finally, in section 7, the primary and secondary indirect effects complete the theory for the CGG2000 geoid model.

The next two sections deal with data and validation of the CGG2000 geoid model. Section 8 gives a brief description of the gravity data and digital elevation models used for the determination of the CGG2000 geoid heights. Section 9 discusses the validation of the CGG2000 geoid model against GPS/Levelling measurements across Canada and the comparison of CGG2000 to latest geoid model for the USA. Finally, the last section constitutes the conclusion and discussion of this paper.

2 Anomalous Potential and Geoid Height: Bruns' Formula

The gravity potential W is created by the gravitational potential V generated by the Earth's masses and by the centrifugal potential Φ induced by the earth's rotation. The gravity potential may be expressed as a sum of a normal gravity potential U generated from a geocentric biaxial ellipsoid spinning with the same angular velocity as the earth and an anomalous potential T. Thus, the gravity potential at a point P on the topography is

$$W_p = U_p + T_p \, . \tag{2.1}$$

The geoid is defined as the equipotential surface of the gravity potential W_g (= constant) of the Earth coinciding with the mean sea level. Thus, the gravity potential on the geoid g is

$$W_g = U_g + T_g \, . \tag{2.2}$$

The determination of a geoid model by Stokes' integral holds only when the anomalous potential is harmonic function outside the geoid. Therefore, no masses should be present above the geoid. Helmert's second condensation method is one approach that satisfies the above condition. It consists of transforming all masses above the geoid (topography and atmosphere) to a condensed layer onto the geoid. For the Helmert approach, the anomalous potential T becomes the sum of the Helmert anomalous potential T^h and the potential of the transformed masses δV. Thus, the gravity potential on the geoid can be expressed as

$$W_g = U_g + T_g^h + \delta V_g \ . \tag{2.3}$$

The potential difference δV between the masses above the geoid and the condensed (c) masses is constituted of the topography (t) and the atmosphere (a), i.e.,

$$\delta V_g = \delta V_g^t + \delta V_g^a = V_g^t - V_g^{ct} + V_g^a - V_g^{ca} \, . \tag{2.4}$$

The normal potential on the geoid U_g can be expanded by the Taylor series and expressed as (Martinec, 1993, 1998):

$$U_g = U_0 + \left.\frac{\partial U}{\partial r}\right|_0 N + \left.\frac{\partial^2 U}{\partial r^2}\right|_0 N^2 + \ldots \tag{2.5a}$$

$$U_g = U_0 - \gamma_0 N + \tau_N \tag{2.5b}$$

where U_0 (= constant) is the normal gravity potential on the reference ellipsoid, γ_0 is the normal gravity on the ellipsoid, N is the geoid-ellipsoid separation (geoid height) and τ_N is the truncation error.

By inserting equation (2.5b) into equation (2.3), it becomes

$$W_g = U_0 - \gamma_0 N + T_g^h + \delta V_g + \tau_N \, . \tag{2.6}$$

Thus, for the Helmert approach, the Bruns formula, which relates the geoid height to the anomalous potential, is given by

$$N = \frac{1}{\gamma_0}(U_0 - W_g + T_g^h + \delta V_g + \tau_N) = \frac{1}{\gamma_0}(\delta W + T_g^h + \delta V_g + \tau_N)\, , \tag{2.7}$$

where δW is the difference between the gravity potential (W_g) at the geoid and the normal gravity potential (U_0) at the surface of the reference ellipsoid. The term T_g^h/γ_0 yields the separation between the co-geoid and the ellipsoid. The following term $\delta V_g/\gamma_0$ is the primary indirect effect (separation between the *geoid* and the co-geoid caused by the condensation of the topography and atmosphere). The last term τ_N/γ_0, which originates from the truncation of the Taylor series in equation (2.5b), has a maximum magnitude of 1 mm (Martinec, 1993).

Finally, we define the gravity potential W_g in relation to an Earth where all topography is removed and where the atmosphere is transformed according to Helmert's second condensation method. We define this modified Earth as "*Bouguer*"; and the superscript *B* expresses it. This "*Bouguer*" Earth can be expressed in potential as

$$W_g = U_g + T_g^B + V_g^t + \delta V_g^a \,. \tag{2.8}$$

This equation will be used in the next section for the purpose of the downward continuation.

3 Boundary Condition for the Helmert Anomalous Potential

Heiskanen and Moritz (1967) give the definition of a gravity disturbance as

$$\delta g_P = g_P - \gamma_P = -\left(\frac{\partial W_P}{\partial n} - \frac{\partial U_P}{\partial n'}\right), \tag{3.1}$$

where *n* and *n'* are the directions of the plumbline and the normal, respectively.

While *Vaníček et al.* (1999) transform directly the masses above the geoid to a condensed layer onto the geoid (i.e., they go directly from the actual Earth to a Helmert Earth), we remove the topography, apply the downward continuation and then restore the topography as a condensed layer. Therefore, the approach goes through the intermediate "*Bouguer*" Earth before the determination of the geoid heights in the Helmert Earth. Therefore, the gravity potential in equation (3.1) is substituted by equation (2.8), but it is defined at the topography. Equation (3.1) can be expressed in relation to the "*Bouguer*" Earth as follows

$$\delta g_P = g_P - \gamma_P = -\left(\frac{\partial (U_P + T_P^B + V_P^t + \delta V_P^a)}{\partial n} - \frac{\partial U_P}{\partial n'}\right). \tag{3.2}$$

Because the two directions of the normal almost coincide and the elevation *h* is reckoned along the normal, equation (3.2) reads:

$$\delta g_P = g_P - \gamma_P = -\left(\frac{\partial (T_P^B + V_P^t + \delta V_P^a)}{\partial h}\right) - \varepsilon_n \,, \tag{3.3}$$

where ε_n is the correction for the substitution of the direction of the normal. The correction can be neglected because it reaches only a few µGal (Cruz, 1986).

Normal gravity γ_P can be expanded by means of γ and its derivatives in a Taylor series at point *0* on the ellipsoid, i.e.,

$$\gamma_P = \gamma_0 + \left.\frac{\partial \gamma}{\partial h}\right|_0 (H+N) + \frac{1}{2}\left.\frac{\partial^2 \gamma}{\partial h^2}\right|_0 (H+N)^2 - \ldots = \gamma_0 - D_\gamma + \left.\frac{\partial \gamma}{\partial h}\right|_0 N + \tau_\gamma \,, \tag{3.4}$$

where D_γ is the normal free-air reduction (normal gradient of gravity time height) to the second order term between the topography surface and geoid. It is defined as follows (Heiskanen and Moritz, 1967)

$$D_\gamma = \frac{2\gamma_0}{a}(1+f+m-2f\sin^2\phi)H - \frac{3\gamma_0}{a^2}H^2 . \tag{3.5}$$

Inserting equation (3.4) into equation (3.3), it becomes

$$\delta g_P = g_P - \gamma_0 + D_\gamma - \left.\frac{\partial\gamma}{\partial h}\right|_0 N + \tau_\gamma = -\left.\frac{\partial T^B}{\partial h}\right|_P - \left.\frac{\partial V^t}{\partial h}\right|_P - \left.\frac{\partial \delta V^a}{\partial h}\right|_P - \varepsilon_n , \tag{3.6}$$

and it can be expressed as:

$$-\left.\frac{\partial T^B}{\partial h}\right|_P = \Delta g_{FA} - \left.\frac{\partial\gamma}{\partial h}\right|_0 N + \left.\frac{\partial V^t}{\partial h}\right|_P + \left.\frac{\partial \delta V^a}{\partial h}\right|_P + \tau_\gamma + \varepsilon_n . \tag{3.7}$$

It corresponds to a gravity disturbance for the "*Bouguer*" Earth. The free-air anomaly is expressed by Δg_{FA}. Because the disturbance is defined at the topography, it must be downward continued to the geoid. Taking spherical approximation of the second term, equation (3.7) can be expressed on the geoid as

$$-\left.\frac{\partial T^B}{\partial r}\right|_g = \Delta g_{FA} - \left.\frac{\partial\gamma}{\partial h}\right|_0 N + \left.\frac{\partial V^t}{\partial r}\right|_P + \left.\frac{\partial \delta V^a}{\partial r}\right|_P + D_{\delta g^B} + \varepsilon_h(T^B) + \varepsilon_h(V^t) + \varepsilon_h(\delta V^a) + \varepsilon_n + \tau_\gamma , \tag{3.8}$$

where the ellipsoidal correction term $\varepsilon_h(T)$ is given by (Jekeli, 1981 and Cruz, 1986)

$$-\frac{\partial T}{\partial h} \cong -\frac{\partial T}{\partial r} - e^2 \sin\phi\cos\phi \frac{1}{r}\frac{\partial T}{\partial \phi} = -\frac{\partial T}{\partial r} + e^2 \sin\phi\cos\phi\ \xi\gamma = -\frac{\partial T}{\partial r} - \varepsilon_h(T) , \tag{3.9}$$

and $D_{\delta g}{}^B$ is the downward continuation of gravity disturbances for the "*Bouguer*" Earth. It is given by

$$D_{\delta g^B} = -\left.\frac{\partial T^B}{\partial r}\right|_g + \left.\frac{\partial T^B}{\partial r}\right|_P . \tag{3.10}$$

Substituting N by equation (2.7) and adding the condensed topographical effect on each side of equation (3.8), it reads

$$-\left.\frac{\partial T^h}{\partial r}\right|_g + \frac{1}{\gamma_0}\left.\frac{\partial\gamma}{\partial h}\right|_0 (\delta W + T_g^h + \delta V_g + \tau_N) = \Delta g_{FA} + \left.\frac{\partial V^t}{\partial r}\right|_P - \left.\frac{\partial V^{ct}}{\partial r}\right|_g + \left.\frac{\partial \delta V^a}{\partial r}\right|_P + D_{\delta g^B} + \varepsilon_n + \varepsilon_h(T) + \tau_\gamma . \tag{3.11}$$

The derivative of the normal gravity along the normal can be expressed as (Jekeli, 1981 and Cruz, 1986)

$$\frac{1}{\gamma_0}\left.\frac{\partial\gamma}{\partial h}\right|_0 \chi \cong -\frac{2}{r}\chi + e^2(3\sin^2\phi - 2)\frac{\chi}{r} = -\frac{2}{r}\chi - \varepsilon_\gamma(\chi) , \tag{3.12}$$

where χ can be substituted by δW, $T_g{}^h$, δV_g and τ_n.

Substituting the two above equations in equation (3.11), it reads

$$-\left.\frac{\partial T^h}{\partial r}\right|_g - \frac{2}{r}T_g^h \cong \Delta g_{FA} + \left.\frac{\partial V^t}{\partial r}\right|_P - \left.\frac{\partial V^{ct}}{\partial r}\right|_g + \left.\frac{\partial \delta V^a}{\partial r}\right|_P + D_{\delta g^B} + \frac{2}{r}(\delta W + \delta V_g + \tau_N) + \mathrm{E} \tag{3.13}$$

or

$$\Delta g_g^h \cong \Delta g_{FA} + \left.\frac{\partial V^t}{\partial r}\right|_P - \left.\frac{\partial V^{ct}}{\partial r}\right|_g + \left.\frac{\partial \delta V^a}{\partial r}\right|_P + D_{\delta g^B} + \frac{2}{r}(\delta W + \delta V_g + \tau_N) + \mathrm{E}\,, \tag{3.14}$$

where

$$\mathrm{E} = \varepsilon_h(T_P) + \varepsilon_\gamma(T_g) + \varepsilon_\gamma(\delta W) + \varepsilon_\gamma(\tau_N) + \varepsilon_n + \tau_\gamma\,. \tag{3.15}$$

Equation (3.13) defines a Helmert gravity anomaly on the geoid.

4 Stokes' Integral

In section 2, we demonstrated that geoid heights could be determined from the Bruns formula. Equation (2.7) can also be expressed as (Heiskanen and Moritz, 1967)

$$N = \frac{\delta W}{\gamma_0} + \frac{R}{4\pi\gamma_0}\int_\Omega \Delta g_g^h\, S(\psi)\, d\Omega + N_{PIE} + \varepsilon_1 + N_{\tau_N}\,, \tag{4.1}$$

where the second term on the right hand side is the Stokes integral, N_{PIE} is the primary indirect effect, ε_l is the ellipsoid correction for the spherical Stokes kernel $S(\psi)$and N_τ is the truncation error. The primary indirect effect contains the contribution from the topography and atmosphere.

Because Helmert gravity anomalies determined from observed gravity contain biases, we remove the long wavelength components of the gravity measurements by subtracting the first l degrees from the Stokes kernel. The long wavelengths up to degree l are added back from a global geopotential model N_l^h in a Helmert Earth. Equation (4.1) becomes

$$N = \frac{\delta W}{\gamma_0} + N_l^h + \frac{R}{4\pi\gamma_0}\int_\Omega \Delta g_g^h\, S^l(\psi)\, d\Omega + N_{PIE} + \varepsilon_1^l + N_{\tau_N}\,, \tag{4.2}$$

where ε_l^l is the ellipsoidal correction above degree l. The spheroidal Stokes kernel $S^l(\psi)$ can be expressed as

$$S^l(\psi) = S(\psi) - \sum_{n=2}^{l}\frac{2n+1}{n-1}P_n(\cos\psi)\,. \tag{4.3}$$

Furthermore, we do not have gravity measurements available over the whole world. Therefore, we split the Stokes integral into two areas: 1) an area σ where we have gravity measurements and 2) an area Ω–σ where we do not have any observed gravity. Ω corresponds to the integration over the whole Earth. Equation (4.2) reads

$$N = \frac{\delta W}{\gamma_0} + N_l^h + \frac{R}{4\pi\gamma_0}\int_\sigma \Delta g_g^h\, S^l(\psi)\, d\Omega + \frac{R}{4\pi\gamma_0}\int_{\Omega-\sigma} \Delta g_g^h\, S^l(\psi)\, d\Omega + N_{PIE} + \varepsilon_1^l + N_{\tau_N} \tag{4.4}$$

Naturally, for evaluating a geoid height, one requires gravity measurements globally. Therefore, a global geopotential model can approximate the actual gravity measurements outside area σ.

5 Spherical Harmonic Model

The gravitational potential is given by

$$V = \frac{GM}{r}\{1 - \sum_{n=2}^{N}\left(\frac{a}{r}\right)^{n}\sum_{m=0}^{n}\left(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda\right)\overline{P}_{nm}(\sin\phi)\} . \tag{5.1}$$

Spherical harmonic expansion of the normal gravitational potential is

$$U^{*} = \frac{GM^{e}}{r}\{1 - \sum_{n=2}^{N}\left(\frac{a^{e}}{r}\right)^{n}\left(\overline{J}_{n0}\right)\overline{P}_{n0}(\sin\phi)\} . \tag{5.2}$$

Therefore, using the disturbing potential and Bruns' formula, one derives

$$N_{L} = \frac{G\delta M}{r_{e}\gamma_{0}} + \frac{GM}{r_{e}\gamma_{0}}\sum_{n=2}^{L}\left(\frac{a}{r_{e}}\right)^{n}\sum_{m=0}^{n}\left((\overline{C}_{nm} - \frac{GM^{e}}{GM}\left(\frac{a^{e}}{a}\right)^{n}\overline{J}_{n})\cos m\lambda + \overline{S}_{nm}\sin m\lambda\right)\overline{P}_{nm}(\sin\phi) \tag{5.3}$$

and

$$\Delta g_{L} = \frac{G\delta M}{r_{e}^{2}} + \frac{GM}{r_{e}^{2}}\sum_{n=2}^{L}\left(\frac{a}{r_{e}}\right)^{n}(n-1)\sum_{m=0}^{n}\left((\overline{C}_{nm} - \frac{GM^{e}}{GM}\left(\frac{a^{e}}{a}\right)^{n}\overline{J}_{n})\cos m\lambda + \overline{S}_{nm}\sin m\lambda\right)\overline{P}_{nm}(\sin\phi) \tag{5.4}$$

where N_L and Δg_L are the geoid height and free-air anomaly up to degree and order L, respectively. The difference between the mass of the equipotential ellipsoid of revolution (M^e) and the mass of and the actual earth (M) is taken into consideration by the first term.

The geoid height N_L, which is determined from the spherical harmonic expansion can also be evaluated from the Stokes integral:

$$N_{L} = \frac{G\delta M}{r_{e}\gamma_{0}} + \frac{R}{4\pi\gamma_{0}(\phi)}\int_{\Omega}\Delta g_{L}\, S(\psi)\, d\Omega + \varepsilon_{2} . \tag{5.5}$$

The ellipsoidal correction ε_2 is a correction for the spherical Stokes kernel. As previously, the geoid height N_L can be determined from the spheroidal Stokes kernel $S^l(\psi)$ of degree l and the integral split into the same two regions

$$N_{L} = N_{l} + \frac{R}{4\pi\gamma_{0}(\phi)}\int_{\sigma}\Delta g_{L}\, S^{l}(\psi)\, d\Omega + \frac{R}{4\pi\gamma_{0}(\phi)}\int_{\Omega-\sigma}\Delta g_{L}\, S^{l}(\psi)\, d\Omega + \varepsilon_{2}^{l} . \tag{5.6}$$

The degree L (>= 360) is significantly higher than the degree l (<= 60). We replace in equation (4.4) the unknown gravity anomalies by gravity anomalies derived from the spherical harmonic model as an approximation. Equation (4.4) reads

$$N \cong \frac{\delta W}{\gamma_0} + N_l^h + \frac{R}{4\pi\gamma_0}\int_{\sigma} \Delta g_g^h \, S^l(\psi)\, d\Omega + \frac{R}{4\pi\gamma_0}\int_{\Omega-\sigma} \Delta g_L \, S^l(\psi)\, d\Omega + N_{PIE} + \varepsilon_1^l + N_{\tau_N} \tag{5.7}$$

where

$$N_l^h = N_l - N_{PIE}^l \, . \tag{5.8}$$

The N^l_{PIE} term is the primary indirect effects on the first l degrees. It allows the transformation of the long wavelengths from the actual Earth to the Helmert Earth. Finally, by subtracting equation (5.6) from equation (5.7), it becomes

$$N \cong \frac{\delta W}{\gamma_0} + N_L + \frac{R}{4\pi\gamma_0}\int_{\sigma} (\Delta g_g^h - \Delta g_L)\, S^l(\psi)\, d\Omega + N_{PIE} - N_{PIE}^l + \varepsilon_1^l - \varepsilon_2^l + N_{\tau_N} \, . \tag{5.9}$$

ε_l^l-ε_2^l represents the ellipsoidal correction due to the residual gravity anomalies above degree and order l.

Because the far zone contribution is determined from a global geopotential model, we want to minimize its contribution by modifying the spheroidal Stokes kernel. Equation (5.9) becomes

$$N \cong \frac{\delta W}{\gamma_0} + N_L + \frac{R}{4\pi\gamma_0}\int_{\psi_0} (\Delta g_g^h - \Delta g_L)\, S^l(\psi,\psi_0)\, d\Omega + N_{PIE} - N_{PIE}^l + \varepsilon_1^l - \varepsilon_2^l + N_{\tau_N} \, . \tag{5.10}$$

The modified spheroidal Stokes kernel is given by (Vaníček and Kleusberg, 1987)

$$S^l(\psi,\psi_0) = S^l(\psi) - \sum_{n=0}^{l} \frac{2n+1}{2} t_n(\psi_0) P_n(\cos\psi) \, , \tag{5.11}$$

where t_n are the modification coefficients and ψ_0 is the size of the integration cap.

6 Mean Helmert anomalies

For the determination of the geoid model for Canada, the Helmert gravity anomalies used into the Stokes integral are actually a mean over a geographical cell. Therefore, the mean Helmert anomaly is defined as

$$\overline{\Delta g}_g^h = \frac{1}{A}\int_{\Omega} \Delta g_g^h(\Omega)\, d\Omega \, , \tag{6.1}$$

where A is the area of the cell.

Because of the limited number of gravity measurements, the mean gravity anomalies are estimated from surrounding observations, which may not lie necessarily within the cell. The proposed approach is based on two concepts: 1) the refined Bouguer anomalies offer a relatively smooth gravity field for interpolation and averaging; and 2) least-squares collocation is an efficient method for gravity prediction. This is the same approach as for GSD95. The refined Bouguer anomaly is defined as (Heiskanen and Moritz, 1967)

$$\Delta g^{rB} = \Delta g_{FA} - 2\pi G\rho H + A_{\psi_0}^t \, , \tag{6.2}$$

where $2\pi G\rho H$ is the Bouguer plate and A^t is the terrain correction within a radius ψ_0 = 50 km.

The mean refined Bouguer anomaly can be determined as

$$\overline{\Delta g}^{rB} = \frac{1}{n}\sum_{i=1}^{n} C_i (\Delta g^{rB}_{j=1,m}) , \tag{6.3}$$

where C is the least-squares collocation operator, n (= 9) is the number of sub-areas within area A and m (=20) is the number of observations (5 nearest observations from each quadrant).

From equation (3.14), we can write the removal of the topography, the restoration of condensed topography and the atmospheric correction as

$$\left.\frac{\partial V^t}{\partial r}\right|_P = B_p^t + A_{\psi_0}^t + A_{\psi_1}^t + A_{\psi_2}^t = B_p^t + STC , \tag{6.4}$$

$$\left.\frac{\partial V^{ct}}{\partial r}\right|_g = B_g^{ct} + A_{\psi_0}^{ct} + A_{\psi_1}^{ct} + A_{\psi_2}^{ct} \cong B_g^{ct} + A_{\psi_1}^{ct} + A_{\psi_2}^{ct} \cong B_g^{ct} + CTE , \tag{6.5}$$

and

$$\left.\frac{\partial \delta V^a}{\partial r}\right|_P = \delta A^a , \tag{6.6}$$

respectively. Terms B_p^t and B_g^{ct} are the attraction of the Bouguer shell and condensed Bouguer shell, respectively. They are given by (Martinec, 1998)

$$B_p^t = -4\pi G\rho H \frac{R^2}{(R+H)^2}(1+\frac{H}{R}+\frac{H^2}{3R^2}) \tag{6.7}$$

$$B_g^{ct} = -4\pi G\sigma \frac{R^2}{(R+H)^2} , \tag{6.8}$$

where σ is the condensed topographical density

$$\sigma = \rho H(1+\frac{H}{R}+\frac{H^2}{3R^2}) . \tag{6.9}$$

The subscripts ψ_0, ψ_1 and ψ_2 to the spherical terrain correction (*STC*) and condensed terrain effect (*CTE*) represent integration areas ranging from 0 to 50 km, 50 km to 3.0 degrees and outside the three-degree cap, respectively. The three zones are evaluated by numerical integration. However, the contribution of the condensed topography on the geoid in the innermost zone $A_{\psi 0}{}^{ct}$ is small. Its contribution to the geoid model reaches 4 mm. However, the difference between the attraction of the Bouguer shell at the topography and the condensed shell at the geoid can contribute up to 5 cm in the Canadian Rocky Mountains.

Therefore, by inserting Eq. (6.3), (6.4), (6.5) and (6.6) into equation (3.14), it yields

$$\overline{\Delta g}_g^h \cong \overline{\Delta g}_{rB} + 2\pi G\rho H_{DEM} + B_p^t - B_g^{ct} + \overline{A}_{\psi_1}^t + \overline{A}_{\psi_2}^t - \overline{A}_{\psi_1}^{ct} - \overline{A}_{\psi_2}^{ct} + \overline{\delta A}^a + \overline{D}_{\delta g^{rB}} + \frac{2}{r}(\delta W + \overline{\delta V}_g + \tau_N) + \mathrm{E} . \tag{6.10}$$

It represents a mean Helmert anomaly on the geoid.

7 Primary and secondary Indirect Effect

Martinec (1993, 1998) gives the formulation for the primary indirect terrain effect (*PITE*) for the spherical approximation:

$$N_{PITE} = -\frac{2\pi G}{\gamma}\rho(\Omega)H^2(1+\frac{2}{3}\frac{H}{R}) \qquad \text{(1}^{\text{st}}\text{ order term)}$$
$$+\frac{G}{\gamma}\int_{\Omega'}[\rho(\Omega')\widetilde{N}(R,\psi,r')\Big|_{r'=R}^{R+H'} - R^2\sigma(\Omega')N(R,\psi,R)$$
$$-\rho(\Omega)\widetilde{N}(R,\psi,r')\Big|_{r'=R}^{R+H} + R^2\sigma(\Omega)N(R,\psi,r')]d\Omega' . \qquad \text{(2}^{\text{nd}}\text{ order term)} \qquad (7.1)$$

The primary indirect topographic effect is evaluated in two zones: inner and outer. The inner zone contribution, which corresponds to a three-degree cap, is evaluated by numerical integration while the integration for the remaining of the world is determined from a spherical harmonic expansion (Novák et al., 2001).

For regions with an ice cap, such as Greenland, the formulation of *PITE* must take into consideration the ice thickness. The density of ice (ρ_{ice}) is 0.917 g/cm^3, which is 2.9 time less than the mean topographical density. Therefore, the formulation of the primary indirect terrain effect at a point on an ice cap with thickness *D* can be approximated by

$$N_{PITE} \cong -\frac{\pi G}{\gamma}[\rho_{ice}H^2(1+\frac{2}{3}\frac{H}{R}) - (\rho_{ice}-\rho_{land})(H-D)^2(1+\frac{2}{3}\frac{H-D}{R})], \qquad (7.2)$$

where *H* is the elevation of the surface above the geoid. Equation (7.2) omits the second order term and the far zone contribution.

The primary indirect atmospheric effect (*PIAE*) is currently judged negligible because its maximum magnitude on the geoid is 6 mm (Vaníček et al., 1998b).

The secondary indirect terrain effect (*SITE*) is determined from the primary indirect terrain effect (*PITE*) (Martinec 1993, 1998), i.e.,

$$\Delta g_{SITE} = \frac{2}{r}\gamma N_{PITE} . \qquad (7.3)$$

On the other hand, the maximum magnitude of the secondary indirect atmospheric effect (*SIAE*) on gravity is 2 μGal (Vaníček et al., 1998). Its contribution is negligible on the geoid.

8 Data and Computation of CGG2000

The CGG2000 geoid model covers most of North America. It includes Canada, the continental USA (including Alaska and and part of Hawaii), Greenland, Iceland, and most of Mexico; however, the model is best determined for Canada. The model has a resolution of 2 minutes of arc in both latitude and longitude and is delimited by the following geographical coordinates: 84°N, 20°N, 170°W and 10°W. The geoid heights of CGG2000 represent the separation between the geocentric reference ellipsoid of GRS80 and the equipotential surface W_0 = (62636855.8 ± 0.5) m^2/s^2 (Burša et al., 1997). The reference system is ITRF97, but it does not correspond to any specify epoch. EGM96 (Lemoine et al. 1998) defines the long wavelengths up to degree l = 30, while coefficients from degree 31 to 360 contribute to the shorter wavelengths of the gravity field outside the six-degree integration cap. It also defines the mass (M) of the actual Earth.

The surface gravity measurements (land and water) are obtained from the following agencies: 690,742 values from Geodetic Survey Division of Natural Resources Canada, 1,477,300 values from U.S. National Geodetic Survey (NGS) and National Imagery and Mapping Agency (NIMA) and 117,086 values from Kort Matryxkelrysen (KMS) in Denmark. The model also includes airborne gravity data for Greenland received from NIMA and KMS. In addition, the shipboard measurements are augmented by a combination of satellite altimetry-derived gravity data determined by *Sandwell and Smith* (v9.2, 1997), KMS v99 (Andersen and Knudsen, 1998), CLS/SHOM v98 and GSFC v2000. All gravity measurements are (or assumed to be) tied to the IGNS71 in a tide-free system.

The Digital Elevation Model (DEM) for Canada is from provincial and federal sources. The DEM for the provinces of British Columbia and Alberta is derived from maps at a scale of 1:20k while the DEM for the province of New Brunswick is obtained from maps at a scale of 1:10k. The elevations have a vertical accuracy better than 10 metres and a horizontal resolution of 25 to 100 metres depending of the topography. The heights for the rest of Canada are obtained from the Canadian Digital Elevation Data (CDED), which are elevations scanned from maps 1:250k with a horizontal resolution of 3 seconds of arc and a vertical accuracy ranging from 30 metres to 150 metres. These new DEM allowed the determination of improved terrain corrections at the gravity measurements in Canada. For the USA (including Alaska), the original version of the Digital Terrain Elevation Data (DTED) is used again in CGG2000 as it was used in GSD95. These elevations are processed following a similar approach as CDED. This DTED version in CGG2000 is of lesser quality than the DEM used currently in the USA geoid model G99SSS (Smith and Roman, 2001). KMS made available to GSD the DEM and ice thickness model for Greenland. Finally, the elevations for the rest of the world are obtained from GTOPO30. We also use the spherical harmonic model TUG-87 (Wieser, 1987) for the far zone contribution of *PITE*.

The mean Helmert gravity anomalies used in determination of the CGG2000 geoid model were evaluated following equation (6.10) with the exception that we omitted the downward continuation of the Bouguer gravity disturbances (eqn. 3.10). UNB and GSD conducted major efforts in the study of the downward continuation using Helmert and Bouguer anomalies, but the results were still unstable. The experimental geoid models that included the downward continuation were statistically worst than models without it when compared to geoid heights derived from GPS and levelling. We even studied and applied the actual gradient of gravity to the mean gravity along the plumbline for the determination of accurate orthometric height. Fortunately, the contribution of the downward continuation to the "Bouguer" Earth is relatively small; however, it is not negligible and should normally be included in a geoid model. GSD, in cooperation with UNB and UofC, will resume their activities related to the study of downward continuation and make it an integral part of the next model.

Even though the effect of topographical density variation on the geoid model could contribute close to 10 cm (Pagiatakis et al., 2000 and Huang et al. 2001), we decided not to include it into the model because it is not taken into consideration in the levelling network yet.

The Stokes integral is solved by 1D-FFT (Haagman et al., 1993). However, the Stokes kernel is modified to filter out the long wavelengths and minimized the contribution outside a six-degree integration cap. The first 30 degrees of EGM96 define the long wavelengths of the CGG2000 geoid model. These selections are based on a series of tests where we tested different degrees of modification (10 to 360), different integration cap-sizes (0.5 to 18 degrees of arc) and different global geopotential models (EGM96, GFZ97 (Gruber et al., 1997) GRIM5-S1 (Lemoine et al., 1999) and PGM2000A (Pavlis et al., 2000)). For the GRIM5-S1 models, which is a satellite only solution to degree 99 and order 95, the far zone contribution to the geoid model outside the integration cap were determined from the EGM96 models.

The experimental geoid models based on different global geopotential models give similar solutions when the modification is equal to or less than degree 30. However, the size of the integration cap plays a significant role in the accuracy of the geoid model. The larger is the integration cap, the larger is the systematic error in the geoid model, but the systematic error diminishes by increasing the degree of modification. Thus, the final selection of the integration cap size and degree of modification is based on minimizing the systematic error without jeopardizing its relative accuracy vis-à-vis geoid heights derived from GPS and levelling. Hopefully, the results from satellite gravity missions will bring accurate gravity information above degree 45 in order to minimize the integration cap size to 4 degrees or less. Thus, the surface gravity measurements would only contribute the high frequency part of the geoid model.

9 Accuracy and validation of CGG2000

An estimated accuracy model at 66% confidence level (1σ) of the CGG2000 geoid model is determined by error propagation and it is depicted in Figure 1. The accuracy for CGG2000 ranges from 0.2 cm to 18 cm. The average error is 2.6 cm with a standard deviation of 1.7 cm. However, the error model does not include the long wavelength errors of the EGM96 geopotential model below degree 30, i.e., wavelengths longer than approximately 600 km. Furthermore, the error model in Figure 1 does not illustrate local systematic errors that might be part of Helmert gravity anomalies. The accuracy of the CGG2000 may be too optimistic for some regions of Canada, but it can be used as an indicator of the quality of the geoid model across Canada.

Currently, the best independent technique for validating geoid model is its comparison to geoid heights derived from GPS ellipsoidal heights and spirit-levelled orthometric heights. To cover as much of the country as possible, a network of GPS surveys conducted by GSD across Canada was assembled and integrated into ITRF97 (epoch 97.0) using the Canadian Base Network (CBN) as the framework (Craymer and Lapelle, 1997). This network is referred to as the GPS Supernet (v3.1a). All GPS surveys from 1986 to 2000 that included occupations of vertical control benchmarks were considered for inclusion in the Supernet (v3.1a).

These surveys used a broad variety of equipment and observing procedures and session lengths (from less than an hour to a few days). Consequently, the computed coordinates were of varying precision, with the 95% confidence interval for ellipsoidal heights ranging mostly from 2 to 10 cm. There were, however, some older surveys which displayed vertical confidence intervals up to a few decimetres, mainly due to the use of single frequency receivers and the mixing of different types of antennae (no estimates of the relative phase centre offsets were available). Parts of the 1989 “GPS on BM” project in British Columbia suffered from these problems. In general, the 95% confidence limit is a good indicator of the precision of the ellipsoidal heights, while the average observation length of each project is a useful indicator of their reliability.

On the other hand, the orthometric heights were determined from a minimum constraint adjustment of geopotential number differences observed after 1981. Benchmark “7629325” in Rimouski, Québec was held fix. In order to increase the number of benchmarks with GPS measurements, we complemented the levelling network by adding all levelling observations since 1972 to the initial adjustment for which the stations were held fix. This approach was chosen in order to minimize the post-glacial rebound effect on the levelling network. Finally, the adjusted geopotential numbers are divided by mean gravity obtained from the 0.0424 factor (Heiskanen and Moritz, 1967). Thus, the orthometric heights are actually the so-called Helmert heights. This adjustment is referred to as Jan01d. It indicates that the mean sea level of the Pacific Ocean in the vicinity of Vancouver is higher than the mean sea level of the Atlantic Ocean next to Halifax by 81 cm. The CGG2000 geoid model indicates a difference of 34 cm between the two coasts. These results are presented in Table 2, which also shows results for previous adjustments of the Canadian primary levelling network and the GSD95 geoid model.

There are a total of 1102 out of 2834 stations in the Supernet (v3.1a) for which heights were available from the Jan01d adjustment. Only twelve heights (1%) were rejected because their discrepancies (h-H-N) were too large vis-à-vis other discrepancies in the immediate neighbourhood. Five of these have ellipsoidal heights with standard deviation larger than 10 cm (95% confidence). For the remaining 7 stations, monument stability must be considered, particularly when reconciling heights, which were measured at different epochs. In some cases the levelling surveys and the GPS surveys have been carried out three decades apart. Thus, l090 stations were available to validate the CGG2000 geoid model.

The height discrepancies between Jan01d heights and GPS-derived orthometric from GSD95 and CGG2000 geoid models as shown in Figures 2a and 2b, respectively. The modification of the Stokes kernel allowed the removal of a significant east-west systematic error in GSD95. This east-west tilt in GSD95 is probably due to systematic errors in the gravity measurements and gravity reductions. The same systematic error would show up in CGG2000, if the standard Stokes kernel was used. The remaining of the systematic error seen in Figure 1b may be from inaccuracy of the global geoid model or systematic error in the levelling. The water level difference between the two coasts may not be as large as 81 cm.

Table 1. Sea surface topography (SST) and water level difference at tide gauges in Vancouver (V), Rimouski (R) and Halifax (H) for different adjustments of the primary levelling network and geoid models. All adjustments use the so-called Helmert heights.

Datum	Sea Surface Topography (cm)			ΔSST (cm)	Comments
	V	H	R[1]	V - H	
Adj. 07-28	-6	-29	-41	+23	Levelling from 1907 to 1928
Adj. 66-71	+159	-46	-41	+205	Levelling from 1966 to 1971
NAVD88	+100	-51	-41	+151	North American Adjustment
GSD95	-60	-41	-41	-19	Geoid Model
Jan98	+54	-49	-41	+103	All Levelling (1907 to 1997)
Jan01d	+34	-47	-41	+81	Levelling from 1982 to 2000
CGG2000	-5	-39	-41	+34	Geoid Model
Jan01d*	-5	-47	-41	+42	Levelling from 1982 to 2000[2]

[1]. Fixed station: (h ITRF97 epoch 97.0 – N CGG2000)*g = 4.2158 m kGal.
[2]. Same as Jan01d but constrained in Rimouski and Vancouver to CGG2000.

New digital elevation models for British Columbia and Alberta contributed significantly in enhancing the geoid model in these two provinces. These new DEM allowed the determination of more accurate terrain corrections to the gravity measurements. In addition, the new Canadian Digital Elevation Data (CDED) available contributed significantly in improving the geoid model in central Québec and Baffin Island.

The discrepancies (h-H-N) for CGG2000 have a standard deviation of 18 cm while it is 41 cm for GSD95. However, the standard deviation for CGG2000 is 8.5 cm after fitting out the systematic biases and tilt by using a four-parameter transformation (Sideris, 1993), which absorbs long wavelength errors. By using the same process, the standard deviation for GSD95 improved to 15 cm, but it remains almost twice as large as the one for CGG2000. These results are summarized in Table 2. If we limit the discrepancies for CGG2000 to GPS measurements forming the CBN adjustment network and levelling lines observed after 1981, the standard deviation improved to 6.7 cm after absorbing the long wavelength errors. By restricting the discrepancies to Western Canada, the standard deviation becomes 4.5 cm.

Table 2. Comparison of different geoid models (GSD95 and CGG2000) to geoid heights derived from GPS ellipsoidal heights (Supernet v.3.1a) and Jan01d primary levelling network.

Geoid Model	Levelling Datum	No	Min (m)	Max (m)	Mean (m)	Std Dev* (m)
GSD95	Jan01d	1090	-1.214	0.404	-0.564	0.410 (0.149)
CGG2000	Jan01d	1090	-0.799	0.238	-0.260	0.179 (0.085)

* The results in parentheses indicate the standard deviation after fitting out systematic biases and tilts by using a four-parameter transformation.

While the goal of CGG2000 is to provide a high-resolution geoid model for Canada, the computation extends over all the conterminous United States, Alaska and Greenland to reduce edge effects. This overlap provides a comparison with the latest purely gravimetric geoid model for USA, G99SSS (Smith and Roman, 2001), developed at the National Geodetic Survey (NGS). The differences (CGG2000 minus G99SSS), ranging from –0.860 m to 1.32 m, are illustrated in Figure 3. These extreme values over the US territory are located in northern Idaho and along the Atlantic coast in the southeast. The large disagreement in the Pacific NorthWest may be related to terrain correction sources while the different technique to combine shipboard gravity data and satellite altimetry data could explain the disagreement along the Atlantic coast. The most significant differences over Canada are due to NGS using an old DEM version.

Unfortunately, the analysis of these two geoid models against 6129 GPS/levelling data across the USA and sea surface height (separation between the ellipsoid and mean sea level) derived from satelliet altimetry could not confirm which model is more accurate. The standard deviation of the discrepancies (h-H-N) for G99SSS (0.217 m), is significantly smaller than for CGG2000 (0.405 m). However, the standard deviation for G99SSS becomes slightly larger than for CGG2000 after removing a tilt by using the four-parameter transformation (see Table 3 and Figures 4a and 4b). The tilt may be coming from the NAVD88 levelling datum, which indicates a mean sea level difference of 1.5 m between Vancouver and Halifax (see Table 1). NGS and GSD will cooperate in solving these large disagreements.

Table 3. Comparison of different geoid models (G99SSS and CGG2000) to geoid heights derived from GPS ellipsoidal heights and NAVD88 orthometric heights.

Geoid Model	Levelling Datum	No	Min (m)	Max (m)	Mean (m)	Std Dev* (m)
G99SSS	NAVD88	6129	-1.127	0.113	-0.524	0.217 (0.163)
CGG2000	NAVD88	6129	-1.905	0.584	-0.731	0.405 (0.135)

* The results in parentheses indicate the standard deviation after fitting out systematic biases and tilts by using a four-parameter transformation.

10 Discussion

The theory described in this paper is the first step towards the realization of a one-centimetre accuracy geoid model for Canada. The paper investigates the formulation of the Bruns equation and gravity anomaly for a Helmert Earth. Furthermore, the “Bouguer” Earth, an Earth where all topographical masses have been removed, is used to ease the downward continuation by creating a smooth gravity field and minimizing its contribution. Even though the downward continuation and the topographical density variation is not considered in this model, the CGG2000 geoid model is the stepping stone of the next generation of geoid models in Canada. This new model is an improvement by a factor of two vis-à-vis the previous geoid model for Canada (GSD95). The main enhancement comes from the modification of the Stokes kernel by filtering out the long wavelength contribution from the surface gravity measurements, which includes systematic errors. As importantly, the new DEM for Canada allowed the determination of more accurate terrain corrections, in particular for the Canadian western Cordillera, and more accurate mean Helmert anomalies.

Future activities in geoid modeling at the centimetre level in Canada are encouraging with progress accomplished in airborne gravity survey from a consortium of Canadian private industry, the University of Calgary and Natural Resources Canada and with the in-coming results from the current and future satellite gravity missions (CHAMP, GRACE and GOCE). In addition, the current project at Natural Resources Canada to create a DEM from maps at 1:50k will allow a better definition of the topography over CDED 1:250k.

The determination of the one-centimetre geoid model is not achieved with this current model, but the theory, data collection (gravity measurements, DEM and topographical density) and computer capability are improving so rapidly that the objective is highly feasible in a reasonable future. Finally, the increasing cooperation between GSD, NGS (USA), KMS (Denmark) and INEGI (Mexico) will culminate to the creation of a unified and accurate geoid model for North America.

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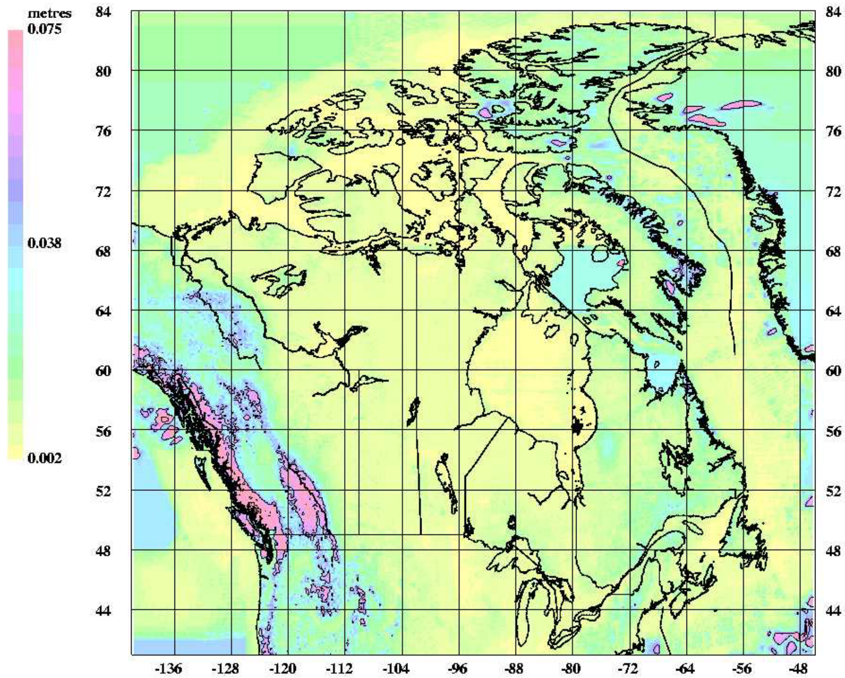
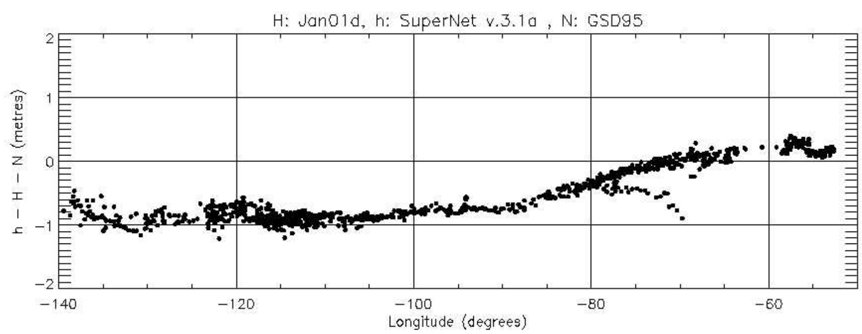
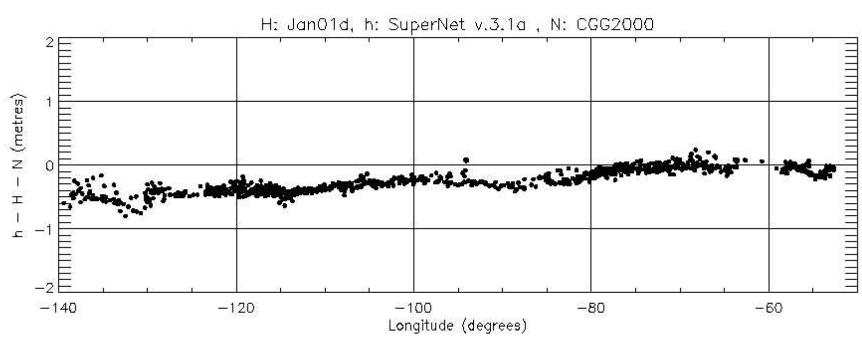


Figure 1: Accuracy of the CGG2000 geoid model above degree 30 (wavelengths longer than approximately 600 km). (Unit: m, C.I.: 5 cm)

Figures 2a: Discrepancies between GSD95 geoid model and geoid heights derived from GPS ellipsoidal heights (Supernet v3.1a) and Jan01d Helmert orthometric heights.





Figures 2b: Discrepancies between the CGG2000 geoid model and geoid heights derived from GPS ellipsoidal heights (Supernet v3.1a) and Jan01d Helmert orthometric heights.

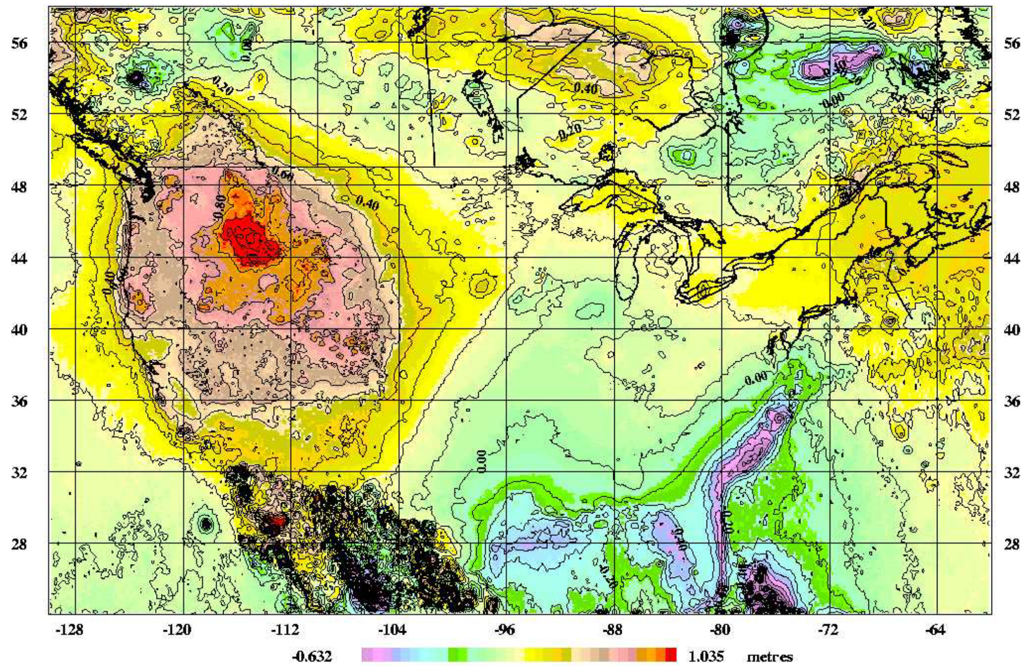
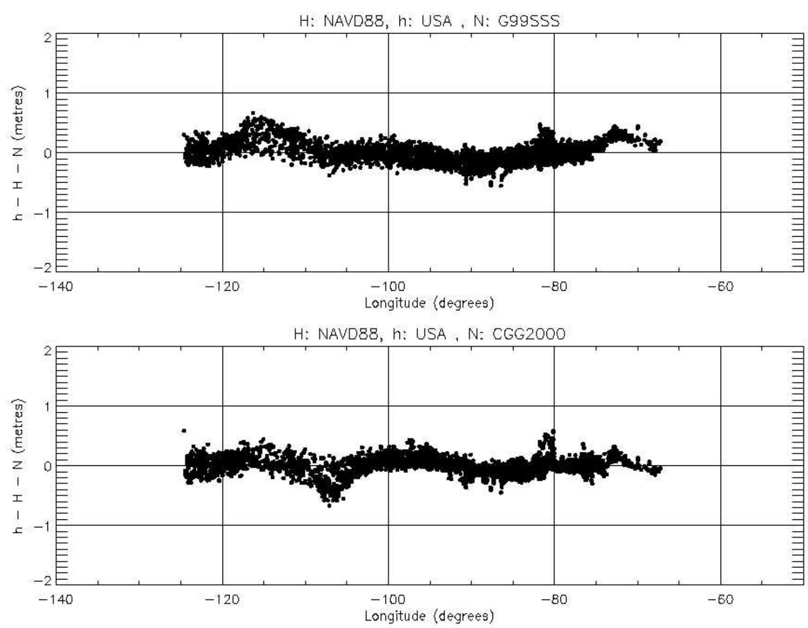


Figure 3: Differences between G99SSS and CGG2000. (Unit: m, C.I.: 10 cm)

Figures 4a: Discrepancies between G99SSS geoid model and geoid heights derived from GPS ellipsoidal heights and NAVD88 Helmert orthometric heights after fitting out systematic biases and tilts by using a four-parameter transformation.



Figures 4b: Discrepancies between CGG2000 geoid model and geoid heights derived from GPS ellipsoidal heights and NAVD88 Helmert orthometric heights after fitting out systematic biases and tilts by using a four-parameter transformation.